

On CP Violation in Minimal Renormalizable SUSY $SO(10)$ and Beyond

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We investigate the role of CP phases within the renormalizable SUSY $SO(10)$ GUT with one 10_H , one $\overline{126}_H$, one 126_H and one 210_H Higgs representations and type II seesaw dominating the neutrino mass matrix. This framework is non trivially predictive in the fermionic sector and connects in a natural way the GUT unification of b and τ Yukawa couplings with the bi-large mixing scenario for neutrinos. On the other hand, existing numerical analysis claim that consistency with quark and charged lepton data prevents the minimal setup from reproducing the observed CP violation via the Cabibbo-Kobayashi-Maskawa (CKM) matrix. We re-examine the issue and find by inspection of the fermion mass sum rules and a detailed numerical scan that, even though the CKM phase preferentially takes values in the second quadrant, the agreement of the minimal model with the data is actually obtained in a non negligible fraction of the parameter space. We then consider a recently proposed renormalizable extension of the minimal model, obtained by adding one chiral 120-dimensional Higgs supermultiplet. We show that within such a setup the CKM phase falls naturally in the observed range. We emphasise the robust predictivity of both models here considered for neutrino parameters that are in the reach of ongoing and future experiments.

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I. INTRODUCTION

Testing neutrino properties is one of the greatest challenges of the contemporary high energy physics. Thanks to the complementary informations provided by solar, atmospheric and reactor based neutrino experiments, we have obtained in the last few years a consistent description of the neutrino phenomenology in terms of the phenomenon of oscillations. The fact that the observed neutrino mass and mixing pattern differs drastically from those typical of the quark sector is remarkable and at the same time intriguing. On the one hand, the typical scale of the neutrino masses (more precisely of neutrino mass differences) lies below the electronvolt scale. On the other hand, large (almost maximal) mixings appear in the lepton sector, at variance with the hierarchical structure of the quark sector. The precise data that we have obtained in the last years on neutrino properties has renewed and spurred the interest on the basic question about the origin of fermion masses and mixings, that remains unanswered within the Standard Model (SM) of electroweak interactions.

The smallness of the neutrino mass may be naturally related to a large Majorana mass scale for the right-handed (RH) components via the seesaw mechanism [1]. This mechanism is naturally embedded in grand unified scenarios. Of particular interest are Grand Unified Theories (GUT) based on $SO(10)$, where all the known fermions, including the RH neutrino components, are contained in three 16-dimensional fundamental spinorial representations. In such a framework the RH Majorana

mass is identified with the left-right symmetry breaking scale.

In the last few years a supersymmetric (SUSY) GUT model based on the $SO(10)$ gauge group has attracted renewed interest for its sharp predictivity of neutrino observables [2, 3]. The model has no more parameters than the Minimal Supersymmetric Standard Model (MSSM) with massive neutrinos and exact R-parity. In the minimal renormalizable setting the model contains three generations of 16_F matter supermultiplets and the following Higgs chiral supermultiplets [4, 5]: 10_H , $\overline{126}_H$, 126_H , and 210_H . The 10_H and $\overline{126}_H$ representations couple to the matter bilinear $16_{Fi}16_{Fj} = (10_S + 120_A + 126_S)_{ij}$ in the superpotential leading to the minimal set of Yukawa couplings needed for a realistic fermion mass spectrum [6] (S, A denote the symmetry property in the generation indices i, j). The $\overline{126}_H$ multiplet also contains a left-handed (LH) Higgs triplet that induces small Majorana neutrino masses via type II seesaw [7]. The 126_H representation is needed in order to preserve supersymmetry from D -term breaking, while the 210_H triggers the $SO(10)$ gauge symmetry breaking and provides the needed mixings among the Higgs supermultiplets.

An attractive property of the model is that when dominance of type II seesaw is invoked [8] the maximality of the atmospheric neutrino mixing can be linked to the $b - \tau$ Yukawa coupling unification [9, 10]. The model features exact R-parity conservation, due to the even congruency class ($B - L = 2$) of the 10 and $\overline{126}$ representations (the 120 representation shares the same property), with relevant implications for cosmology and proton decay [11, 12, 13, 14, 15, 16, 17, 18].

In a previous paper [19] we studied in detail the implications of the mass sum rules of the model on the neutrino mass parameters. We showed that in the case of CP conserving Yukawa couplings (advocating for the

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sake of the discussion the soft supersymmetry breaking sector as the source of the observed CP violation), when $2\text{-}\sigma$ ranges for the quark masses and mixings are considered, the model is consistent with all lepton data at the level of the present accuracy. On the other hand, when the analysis is limited to $1\text{-}\sigma$ ranges the model shows clear tensions in reproducing the observed leptonic spectrum. In particular, the electron mass is reproduced by an extreme fine tuning among quark parameters, while the solar mixing angle θ_{12} is predicted too close to maximal, and the deviation of the atmospheric mixing θ_{23} from maximal is too large. The shortcomings of the model were previously emphasized in the literature [20, 21], albeit considering a too limited range of uncertainties in the input data.

In the same paper we worked out a simple renormalizable extension of the SUSY $SO(10)$ model by including one 120_H supermultiplet to complete the allowed Yukawa interactions. On the basis of analytic arguments and numerical analysis we showed that a small 120_H contribution to the Yukawa potential allows for a substantial improvement on the fit of the lepton mixings even at the $1\text{-}\sigma$ level. At the same time, the set of the model parameters remains overconstrained by the input data, and the framework provides non-trivial outcomes in the neutrino sector.

When complex Yukawa couplings are taken into account, recent analyses [22, 23, 24] show that the fit of the lepton mixings may improve, but the Cabibbo-Kobayashi-Maskawa (CKM) phase is forced to the second or third quadrant by the electron mass fit, thus requiring significant contributions to CP violation from other sources. Non-renormalizable operators are invoked in ref. [23], while in ref. [24] a 120 dimensional Higgs extension is considered (with an additional parity symmetry), in order to restore the agreement with the data. CP violation in a similar SUSY $SO(10)$ framework with a $U(2)$ family symmetry was discussed in ref. [25].

In this paper we reconsider the study of the fermion mass and mixing patterns in the minimal renormalizable SUSY $SO(10)$ model with complex Yukawa couplings. We comment on the simple analytic argument that justifies the CKM phase taking preferentially values in the second or third quadrant and show, by a careful numerical treatment of the fermion mass rules, that the minimal model exhibit no dramatic tension among quark, charged leptons data and CKM CP violation. The fit of neutrino data shows some tension with the value of the strange quark mass which is required to be large in order to reproduce the recently increased lower limit on the solar neutrino mixing. Nevertheless the minimal framework does remain a viable GUT candidate¹. In particular, we

emphasize the sharpness and the robustness of the prediction of the U_{e3} lepton mixing, that is bound to be non-vanishing and within the reach of planned neutrino experiments.

We conclude the discussion by considering the renormalizable 120_H extension of the minimal setting proposed in ref. [19]. While the authors of ref. [24], in order to sensibly reduce the number of parameters, impose an additional parity symmetry that makes all Yukawa couplings hermitian, in ref. [19] it is assumed that the 120_H induced contributions to fermion masses are a perturbation of the minimal scenario (from percent to 0.1 percent level). It was shown that the additional (antisymmetric) Yukawa interaction leads to a dramatic improvement on the fit of the fermion masses and mixings, while maintaining, as a perturbation, the predictivity of the minimal framework. Here we address CKM CP violation and show that in the extended model the agreement with the data is easily obtained, while maintaining a robust non vanishing lower bound for the U_{e3} entry of the lepton mixing.

II. FERMION MASSES AND MIXING IN THE MINIMAL RENORMALIZABLE MODEL

Henceforth, unless otherwise stated, we will follow and refer to the notation of ref. [19]. When type II seesaw is considered, the mass matrices for the SM fermions in the minimal renormalizable SUSY $SO(10)$ scenario are given by [6, 27]

$$\begin{aligned} M_u &= Y_{10}v_u^{10} + Y_{126}v_u^{126}, \\ M_d &= Y_{10}v_d^{10} + Y_{126}v_d^{126}, \\ M_l &= Y_{10}v_d^{10} - 3Y_{126}v_d^{126}, \\ M_\nu &= Y_{126}v_T^{126}, \end{aligned} \quad (1)$$

where Y_{10} and Y_{126} are complex symmetric 3×3 matrices, $v_{u,d}^{10,126}$ denote the VEVs of the components of the 10_H and $\overline{126}_H$ multiplets that enter the light Higgs doublets of the MSSM, and v_T^{126} is the tiny induced VEV of the LH triplet component in 126_H . These relations can be translated into the following GUT scale sum rules for the charged lepton and (Majorana) neutrino mass matrices [22]:

$$k\tilde{M}_l = \tilde{M}_u + r\tilde{M}_d \quad M_\nu \propto M_l - M_d \quad (2)$$

where k and r are in general complex $\mathcal{O}(1)$ functions of $v_{u,d}^{10,126}$, and the tilded matrices are normalized to their maximal eigenvalue. Let us first rescale by a global phase

¹ Due to the large chiral super multiplets present in the model the running gauge coupling diverges shortly above the GUT scale. This may call for additional unknown physics to enter below the

Planck scale [26] or for an effective gravitational scale to enter near the GUT scale. Such a discussion is beyond the scope of this paper in as much as perturbativity is maintained up to the GUT scale.

eq. (2) such that r becomes real ($r = -|r|e^{i\phi_r}$) and define $k' = ke^{-i\phi_r}$. Since all the mass matrices are symmetric they can be diagonalized by means of unitary transformations $M_x = U_x^T D_x U_x$. Taking into account that $U_u^T U_d^* \equiv V_{CKM}^0 = P_u V_{CKM} P_d$, where $P_u = \text{diag}(e^{\frac{1}{2}i\alpha_1}, e^{\frac{1}{2}i\alpha_2}, e^{\frac{1}{2}i\alpha_3})$ and $P_d = \text{diag}(e^{\frac{1}{2}i\beta_1}, e^{\frac{1}{2}i\beta_2}, 1)$ parametrize the re-phasing of the quark mass matrices necessary to bring the CKM matrix in the standard form (denoted by V_{CKM} with one CP-violating phase, δ_{CKM}) and lead to positive eigenvalues, we may rewrite the sum-rules in the M_d -diagonal basis as follows

$$\begin{aligned} k' \tilde{M}'_l &= V_{CKM}^0{}^T \tilde{D}_u V_{CKM}^0 - |r| \tilde{D}_d \\ k' \tilde{M}'_\nu &\propto V_{CKM}^0{}^T \tilde{D}_u V_{CKM}^0 - |r| \tilde{D}_d - e^{i\omega} \left| k' \frac{m_b}{m_\tau} \right| \tilde{D}_d \end{aligned} \quad (3)$$

The primed matrices in eq. (3) are given by $M'_x \equiv U_d^* M_x U_d^\dagger$, while ω accounts for the phase of k' and the sign of m_b/m_τ . The factors $\frac{1}{2}$ in definitions of phases of $P_{u,d}$ are chosen to maintain compatibility with the notation used in ref. [22].

One can now exploit the information contained in eq. (3) by plugging into the charged lepton mass formula all the parameters that are known (by running the data up to the GUT scale), namely the ratios of quark masses in $\tilde{D}_u \equiv \text{diag}(|m_u/m_t|, |m_c/m_t|, 1)$, $\tilde{D}_d \equiv \text{diag}(|m_d/m_b|, |m_s/m_b|, 1)$, the CKM mixing angles and the CP-violating phase δ_{CKM} and vary them within their experimental ranges. The remaining 8 real parameters $|r|$, $|k'|$, $\text{Arg}(k')$, α_i , β_j , that appear in the first mass sum-rule, in principle arbitrary are varied over their allowed domains. In spite of the many parameters, the charged lepton mass sum rule (3) is overconstrained and some fine tuning is needed to reproduce the lighter eigenvalues. Having obtained an allowed solution of the charged lepton mass sum rule, the neutrino masses and mixings are then sensitive to the sign of m_b/m_τ and the phase of k' (ω). Since a negative interference between the 3-3 entries of the M_l and M_d matrices is needed to obtain a large atmospheric mixing angle, the phase ω becomes strongly correlated to quark phases. As a consequence this framework is highly constrained in the neutrino sector as well and determines characteristic correlations among the neutrino parameters.

In the light of the above remarks it is undoubtedly intriguing that this scenario is shown to fit the main features of the SM data even when Yukawa phases are neglected [19, 20]. As we have already mentioned, in such a case, when considering 1- σ uncertainties in the input parameters, a rather stringent lower bound for the solar mixing angle $\sin^2 2\theta_{12} \gtrsim 0.85$ appears and the atmospheric mixing angle can hardly be maximal. In addition one finds a lower bound for the $|U_{e3}|$ parameter $|U_{e3}| \gtrsim 0.15$, just at the value of the present 90% C.L. experimental upper bound $|U_{e3}| \lesssim 0.15$. While the constraints on the solar and atmospheric mixings are substantially relaxed when considering 2- σ uncertainties in the input data, the lower bound on $|U_{e3}|$ is robust and

represents a characteristic feature of the model as discussed in [19, 20].

Before proceeding to the discussion of numerical results let us review some issues related to the fit of the fermion spectrum in the case of complex Yukawa couplings.

A. δ_{CKM} and the electron mass formula

When complex Yukawa couplings are considered, it is claimed that a successful fit of the electron mass forces the CKM phase to take values in the second or third quadrant [22, 23, 24], thus requiring an extension of the model to recover the measured CP violation.

The argument that supports the numerical outcome is based on the approximate formula for the electron mass eigenvalue that can be obtained from (3) using the hierarchical properties of the quark mass matrices in the right-hand side (RHS):

$$\begin{aligned} |k' \tilde{m}_e| e^{i\phi_1} &= -|r| e^{i\beta_1} F_d \lambda^4 + e^{i\alpha_2} F_c \lambda^6 \\ &- A^2 \Lambda^2 e^{i\alpha_3} \lambda^6 \frac{|r|}{e^{i\alpha_3} - |r|} + \mathcal{O}(\lambda^7) \end{aligned} \quad (4)$$

Here $\Lambda \equiv (1 - \rho - i\eta)$ where ρ and η are the Wolfenstein CKM parameters, while $F_d \equiv \frac{m_d}{m_b}/\lambda^4$, $F_c \equiv \frac{m_c}{m_t}/\lambda^4$ are $\mathcal{O}(1)$ factors. Fitting the normalized electron mass (with a typical magnitude of the order of $\mathcal{O}(\lambda^5)$) amounts to compensating the dominant λ^4 term in the RHS by other terms therein, the only possible one at the given order of expansion being that proportional to Λ^2 . In turn this amounts to constraining the size of the CKM phase. The CKM phase is encoded in ρ and η as $e^{i\delta_{CKM}} s_{13} = A\lambda^3(\rho + i\eta)$. The typical values of ρ and η for δ_{CKM} in the physical region are centered around $\rho \sim 0.21$ and $\eta \sim 0.34$. Since the parameter $|\Lambda|^2 = (1 - \rho)^2 + \eta^2$ is maximized for $\rho < 0$ (in refs. [19, 20] the CP conserving case $\eta = 0$ was considered), the fit of the electron mass in formula (4) seems to strongly disfavor the CKM phase in the first quadrant [22].

On the other hand, one should be careful in claiming the relevance of “subleading” terms in a truncated expansion. A detailed inspection of the $\mathcal{O}(\lambda^7)$ terms in eq. (4) shows that λ is not a faithful expansion parameter, in that some cofactors, not necessarily dependent on Λ , can become accidentally large (a small denominator in the λ^6 term is an example). Therefore a larger number of “subleading” terms in eq. (4) may contribute on top of the $\mathcal{O}(\lambda^6)$ term, and the scan over the complex phases must be very detailed not to miss such a solutions (we stress that in the minimal model the electron mass is nevertheless the outcome of a fine tuning; what we are here discussing is the extent). As an example let us consider the $\mathcal{O}(\lambda^7)$ term

$$\mathcal{O}(\lambda^7) \sim \frac{A^4 \Lambda^2 |r| e^{i(2\alpha_3 - \beta_2)}}{F_s (e^{i\alpha_3} - |r|)^2} \lambda^7 + \dots, \quad (5)$$

where $F_s \equiv \frac{m_s}{m_b}/\lambda^3$. Since for typical values of r one obtains $(1 - |r|)^2 \approx \lambda^2$, the denominator can be small enough to lead to an important correction to the $\mathcal{O}(\lambda^6)$ term. Notice that a small m_s favors as well the needed destructive interference in eq. (4), as emphasized in ref. [23].

In our numerical analysis we pay particular attention to the quality of the parameter scan in those regions that may lead to departures from the expectations based on the size of Λ in eq. (4). Once an approximate solution of the charged lepton mass sum-rule is found (at the few percent level) a detailed analysis is performed in the neighbor parameter space by linearizing the mass relations. Such a procedure improves dramatically the convergence of the numerical code, revealing solutions that would escape the original scan, unless one performs it with extremely high granularity and huge demand of computing power. Indeed, such an improved numerical analysis shows many solutions of the charged lepton mass formula emerging in parts of the parameter space where the cancelation among the leading terms in eq. (4) is not as effective. We discuss and show our numerical results in the next subsection.

The sensitivity of the electron mass fit to subleading terms in eq. (4) is crucial in the case of the extended model considered in ref. [19] with an additional 120-dimensional chiral super-multiplet. As we shall see in Section III, even for typical magnitudes of the 120_H Yukawa contributions to fermion masses as low as 0.1% of those coming from the 10 and 126 dimensional Higgs multiplets, the role of the Λ^2 term in eq. (4) is easily screened by the 120_H induced terms, thus lifting the bias on the CKM phase.

B. Numerical results

In the first stage we performed a detailed fit of the relations (3). We use GUT scale input data for quark and leptons as derived in ref. [28] via two-loop renormalization group analysis. Table I shows a sample of quark and charged lepton parameters and their uncertainties, for a given choice of the supersymmetric threshold scale M_S and $\tan\beta$. A supersymmetric threshold scale $M_S \simeq 1$ TeV and $\tan\beta = 10$, 55 will be considered as typical values for the MSSM gauge coupling unification underlying the one-step $SO(10)$ breaking scenario here considered. The GUT relations between fermion masses and Yukawa couplings depends on the vacuum expectation values (VEV). Henceforth we follow the $\overline{\text{MS}}$ prescription adopted in ref. [28], to which we refer the reader for details.

We consider 90% C.L. ranges for all input parameters. With respect to the data used in refs. [28, 29] we account for larger uncertainties in the masses of the light u , d quarks and update the range of m_s . While m_u plays a subleading role in the mass sum rules, a light m_d favors the reproduction of the electron mass eigenvalue, as we discussed in the previous subsection. The values of m_u

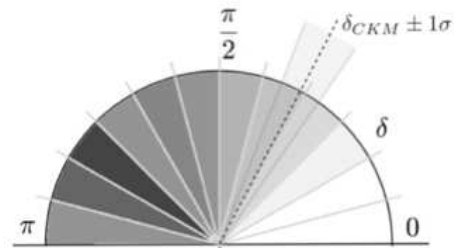
TABLE I: Sample of GUT-scale values for quark masses and mixings and charged lepton masses. The $1\text{-}\sigma$ data are shown for a GUT scale $M_G = 2 \times 10^{16}$ GeV, a supersymmetric threshold scale $M_S = 1$ TeV and $\tan\beta = 10$ [28, 29, 30]. We use the results of ref. [28] up to the u , d and s quark masses whose central values and uncertainties are updated according to the discussion in the text. In our analysis we scan over 90% C.L. ranges (1.64σ) of the input parameters.

m_e	0.3585 MeV
m_μ	$75.67^{+0.06}_{-0.05}$ MeV
m_τ	$1292.2^{+1.3}_{-1.2}$ MeV
m_d	$1.5^{+0.42}_{-0.23}$ MeV
m_s	$29.94^{+4.30}_{-4.54}$ MeV
m_b	$1.06^{+0.14}_{-0.08}$ GeV
m_u	$0.72^{+0.13}_{-0.14}$ MeV
m_c	$210.32^{+19.00}_{-21.22}$ MeV
m_t	$82.43^{+30.26}_{-14.76}$ GeV
$\sin\phi_{12}$	0.2243 ± 0.0016
$\sin\phi_{23}$	0.0351 ± 0.0013
$\sin\phi_{13}$	0.0032 ± 0.0005
δ_{CKM}	$60^\circ \pm 14^\circ$

and m_d given in Table I correspond to 4.88 ± 0.57 MeV and 9.81 ± 0.65 MeV respectively at 1 GeV [29]. The present ranges for the $\overline{\text{MS}}$ running masses at 1 GeV of the up and down quarks are 2 to 5.4 MeV and 5.4 to 10.8 MeV respectively [30]. Accordingly, we will allow for values of m_d at the GUT scale as low as 0.7 MeV. As for the strange quark mass an up to date range, which includes the lattice evaluations, is $m_s(2 \text{ GeV}) = 110 \pm 20$ MeV, corresponding at the GUT scale to 23 ± 6 MeV.

The complex phases $\alpha_{2,3}$, $\beta_{1,2}$ are sampled in the whole range $[0, 2\pi)$. The phase ω shows then a correlation to the quark phases as a consequence of the tight relation between large atmospheric and $b - \tau$ Yukawa unification, which involves a partial cancelation among the terms in the RHS of the neutrino mass sum rule. Since the re-

FIG. 1: The relative density spectrum of allowed solutions for the charged lepton masses is shown as a function of the CKM phase δ . Although there appear a preference for the quadrants with $\rho < 0$, there exists a significant number of solution with δ_{CKM} in the physical region. Quantitatively the ratio of the solution density in the preferred area (the darkest slice) to that in the $1\text{-}\sigma$ range is about 7 to 1.



duced up-quark mass \tilde{m}_u is by far the smallest parameter in the mass relations, α_1 does not play any relevant role.

In Fig. 1 we display the density spectrum of the solutions of the charged lepton mass formula in eq. (4) as a function of δ_{CKM} in the interval $[0, \pi)$.

As one sees the relative density of solutions of the charged lepton sum rule is far from being negligible even when considering δ_{CKM} in the $1\text{-}\sigma$ range, contrary to the claims in refs. [22, 23]. To make this statement quantitative the ratio of the solution density in the most preferred area (the darkest slice in the second quadrant) to that in the $1\text{-}\sigma$ δ_{CKM} range is about 7 to 1.

As far as neutrino parameters are concerned, the solar mixing angle shows no longer the tight lower bound present in the CP-conserving case, namely $\sin^2 2\theta_{12}|_{CP=0} > 0.85$ (for $\tan\beta = 10$) [19]. On the other hand, the experimental improvement on the allowed values for the solar mixing sharpens a tension with the strange quark mass.

FIG. 2: A density plot of $\sin^2 2\theta_{12}$ is shown as a function of $\sin^2 2\theta_{23}$ in the minimal renormalizable SUSY $SO(10)$ model with complex Yukawa couplings for $\tan\beta = 10$. The solid contour encloses the experimentally allowed region at the 90% C.L. The dark area corresponds to the solutions that are consistent with all neutrino data.

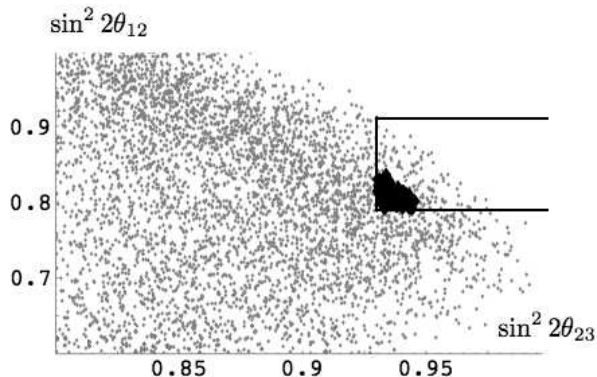


Fig. 2 shows the area in the solar and atmospheric mixing angles that is consistent with the ratio of neutrino mass square differences. While the latter (Fig. 3) bounds this area from above, we find that the 90% C.L. lower bound on the solar mixing requires a strange quark mass above 30 MeV at the GUT scale, that corresponds to $m_s(2 \text{ GeV}) > 140 \text{ MeV}$ (the solutions in the allowed region span a GUT scale strange quark mass in the 30–34 MeV range). Should the experimental value for the solar mixing angle settle above the present central value, it would represent a serious shortcoming of the minimal $SO(10)$ framework. The same conclusion applies to a maximal atmospheric mixing.

The lower bound for the $|U_{e3}|$ parameter is relaxed compared to the CP conserving case, although not as dramatically as for the solar angle. As Fig. 4 shows, for $\tan\beta = 10$ the constraint $|U_{e3}| \geq 0.15$ found in the CP conserving setting (see the discussion in ref. [19]) is

FIG. 3: A density plot of $|\Delta m_{\odot}^2/\Delta m_A^2|$ is shown as a function of $\sin^2 2\theta_{23}$ in the minimal renormalizable SUSY $SO(10)$ model with complex Yukawa couplings for $\tan\beta = 10$. The solid contour encloses the experimentally allowed region at the 90% C.L. The dark area corresponds to the solutions that are consistent with all neutrino data.

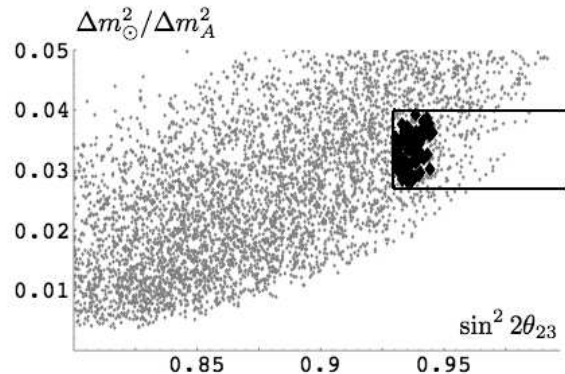
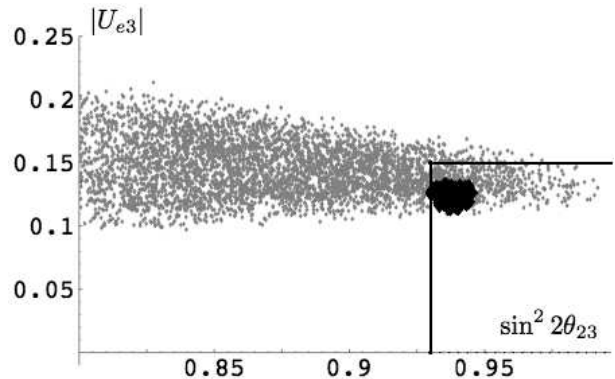


FIG. 4: A density plot of $|U_{e3}|$ is shown as a function of $\sin^2 2\theta_{23}$ in the minimal renormalizable SUSY $SO(10)$ model with complex Yukawa couplings for $\tan\beta = 10$. The solid contour encloses the experimentally allowed region at the 90% C.L. The dark area corresponds to the solutions that are consistent with all neutrino data.



lowered to about $|U_{e3}| \geq 0.1$. In the case of $\tan\beta = 55$ we find at the level of accuracy of our numerical analysis consistent solutions at the 95% C.L. in the neutrino data. On the other hand, the lower bound on $|U_{e3}|$ remains unaffected. The persistence of such a non-vanishing lower bound is a clear signature of the tight correlation between lepton and quark Yukawa couplings in this framework, that makes $|U_{e3}| \simeq O(\lambda)$ [22].

In order to give an explicit numerical example we may consider the following (GUT scale) values for the quark masses [28]

$$\begin{aligned} m_u &\sim 0.57 \text{ MeV} & m_d &\sim 0.73 \text{ MeV} \\ m_c &\sim 235.7 \text{ MeV} & m_s &\sim 31.3 \text{ MeV} \\ m_t &\sim 90.0 \text{ GeV} & m_b &\sim 1.19 \text{ GeV} \end{aligned}$$

together with the CKM parameters

$$\begin{aligned} \sin \phi_{12} &\sim 0.2253 & \sin \phi_{23} &\sim 0.0331 \\ \sin \phi_{13} &\sim 0.0035 & \delta_{CKM} &\sim 75^\circ. \end{aligned}$$

For

$$\begin{aligned} \alpha_1 &\sim 144^\circ & \beta_1 &\sim 216.8^\circ \\ \alpha_2 &\sim 142^\circ & \beta_2 &\sim 224.6^\circ \\ \alpha_3 &\sim 1.2^\circ & \omega &\sim -0.2^\circ \\ |r| &\sim 0.748 & |k'| &\sim 0.256 \end{aligned}$$

one obtains the following charged lepton mass matrix (normalized to the τ mass):

$$100 k' \tilde{M}'_l \approx |k'| \times \begin{pmatrix} -0.146 + 0.004i & -0.134 + 0.016i & 0.35 + 2.862i \\ -0.134 + 0.016i & -6.985 - 0.197i & 5.321 - 11.506i \\ 0.35 + 2.862i & 5.321 - 11.506i & 97.846 + 8.567i \end{pmatrix}$$

The corresponding (GUT-scale) charged lepton masses are all within their 90% C.L. ranges: $m_e = 0.3585$ MeV, $m_\mu = 75.62$ MeV and $m_\tau = 1294.0$ MeV. The neutrino mass matrix is then given by

$$100 \tilde{M}'_\nu \propto \begin{pmatrix} -0.203 + 0.004i & -0.134 + 0.016i & -0.339 + 2.863i \\ -0.134 + 0.016i & -9.404 - 0.224i & 5.366 - 11.485i \\ -0.339 + 2.863i & 5.366 - 11.485i & 5.855 + 8.952i \end{pmatrix}$$

Keeping into account that the absolute mass scale is set by the VEV of the LH triplet in $\overline{126}_H$, one finds neutrino mass ratios and mixings ($\sin^2 2\theta_{12} = 0.82$, $\sin^2 2\theta_{23} = 0.93$, $|U_{e3}| = 0.11$, $\Delta m_\odot^2 / \Delta m_A^2 = 0.027$) that are within the present 90% C.L. experimental data.

The comparison with the data in Table II must account for the running of the parameters from the GUT scale to the weak scale. For normal hierarchy with $m_1^2/m_2^2 \ll 1$ (which is generally the case in the setup here considered) the effects of running of the neutrino mixings and the Δm^2 ratios down to the weak scale are mild [32]. In particular, $|U_{e3}|$ is very stable, with corrections below 1% for the whole range of $\tan \beta$ considered. The variation of the atmospheric angle is small as well, remaining below one percent for $\tan \beta = 10$ and at the percent level for $\tan \beta = 55$. The largest corrections appear for the solar angle at large $\tan \beta$: several percents for $\tan \beta = 55$. Both solar and atmospheric mixing angles grow when approaching the weak scale.

As far as the leptonic CP phases are concerned, the Dirac phase δ_{PMNS} turns out to be generally small ($< 15^\circ$), while the two neutrino Majorana phases $\varphi_{1,2}$ show an approximate 180° correlation. In the example reported one finds $\delta_{PMNS} = 4^\circ$, $\varphi_1 = 10^\circ$, $\varphi_2 = 191^\circ$.

We conclude that the minimal SUSY $SO(10)$ GUT, when complex Yukawa couplings are taken in their generality, is not ruled out by present data on the quark and

TABLE II: Presently allowed values (90% C.L.) for neutrino mixing and mass parameters [31].

$0.79 \lesssim \sin^2 2\theta_{12} \lesssim 0.91$		
$0.93 \lesssim \sin^2 2\theta_{23}$		
$ \sin \theta_{13} \lesssim 0.15$		
$2.7 \times 10^{-2} \lesssim \frac{\Delta m_\odot^2}{\Delta m_A^2} \lesssim 4.0 \times 10^{-2}$		

leptons textures. On the other hand, due to a raising tension with the strange quark mass a large solar mixing and a maximal atmospheric angle can hardly be accommodated. The non-vanishing lower bound for $|U_{e3}|$ is a robust prediction of the model that falls within the reach of the planned long-baseline neutrino experiments (a discussion on proton decay and lepton flavor violation processes within this framework is presented in refs. [17, 23]).

III. THE MODEL WITH QUASI-DECOUPLED 120-DIMENSIONAL HIGGS REPRESENTATION

In the second part of this paper we consider the case of complex Yukawa couplings in a simple extension of the minimal renormalizable $SO(10)$ proposed in ref. [19]. The minimal setting is enlarged to include a 120_H chiral super multiplet to which the $16_F \times 16_F$ matter bilinear couples. At variance with ref. [24] no a-priori restrictions are imposed on the form of the Yukawa couplings. On the other hand, in ref. [19] the 120_H contributions to the fermion masses are assumed to be two to three orders of magnitude smaller than those induced by 10_H and $\overline{126}_H$. This can be seen as a consequence of a partial decoupling of the 120_H multiplet and/or a small Yukawa coupling. In both cases the setup remains stable under quantum corrections and it is (technically) natural. It is then shown that, due to the different symmetry property of the 120_H Yukawa coupling, the related contributions to the mass matrices affect the neutrino mixing angle predictions in such a way to improve substantially the agreement with the data, even at the $1\text{-}\sigma$ level. The model does not lose its predictivity and in particular the sharp prediction for the U_{e3} mixing angle remains.

In this paper we discuss how the outcomes of the non-minimal setting change by switching on CP violation in the Yukawa sector. For a detailed discussion of the extended setup we refer the reader to ref. [19]. We comment here on the features that differ from the real Yukawa case.

Once the left-handed doublets contained in 10_H , $\overline{126}_H$ and 120_H acquire a vacuum expectation value (VEV), the contributions to the quark and lepton mass matrices are generalized to

$$\begin{aligned} M_u &= Y_{10} v_u^{10} + Y_{126} v_u^{126} + Y_{120} v_u^{120}, \\ M_d &= Y_{10} v_d^{10} + Y_{126} v_d^{126} + Y_{120} v_d^{120}, \end{aligned} \quad (6)$$

$$\begin{aligned} M_l &= Y_{10} v_d^{10} - 3Y_{126} v_d^{126} + Y_{120} v_l^{120}, \\ M_\nu &= Y_{126} v_T^{126}. \end{aligned}$$

While Y_{10} and Y_{126} are complex symmetric matrices, Y_{120} is a complex antisymmetric matrix. As discussed in ref. [19], v_x^{120} are taken to be suppressed with respect to v_x^{10} and v_x^{126} by two to three orders of magnitude (alternatively the Yukawa coupling should provide the required suppression). In such a case the Y_{120} -proportional terms in (6) can be treated as perturbations of the minimal model results. This allows us to maintain the successful leading order features of the model while treating the multi-parameter problem via a perturbative approach.

The generalized sum rules for the charged lepton and neutrino mass matrices then read

$$\begin{aligned} k\tilde{M}_l &= \tilde{M}_u + r\tilde{M}_d + Y_{120}(k\varepsilon_l - \varepsilon_u - r\varepsilon_d), \\ \tilde{M}_\nu &\propto \tilde{M}_l - Y_{120}\varepsilon_l - \frac{m_b}{m_\tau}(\tilde{M}_d - Y_{120}\varepsilon_d) \end{aligned} \quad (7)$$

where

$$\varepsilon_u \equiv \frac{v_u^{120}}{m_t}, \quad \varepsilon_d \equiv \frac{v_d^{120}}{m_b}, \quad \varepsilon_l \equiv \frac{v_l^{120}}{m_\tau}.$$

As before, one can rotate away the phase of r and diagonalize all the quark mass matrices by means of *biunitary* transformations $\tilde{M}_x = V_x^R \tilde{D}_x V_x^{L^T}$. The sum rules in eq. (7) can be then recast as follows:

$$\begin{aligned} k'V_d^{R\dagger} \tilde{M}_l V_d^{L*} &= V_d^{R\dagger} V_u^R \tilde{D}_u V_{CKM}^0 - |r|\tilde{D}_d \\ &\quad + Y_{120}'(k'\varepsilon_l - e^{-i\phi_r}\varepsilon_u + |r|\varepsilon_d) \\ k'V_d^{R\dagger} M_\nu V_d^{L*} &\propto k'V_d^{R\dagger} \tilde{M}_l V_d^{L*} - Y_{120}'k'\varepsilon_l \\ &\quad - |k'|e^{i\omega} \left| \frac{m_b}{m_\tau} \right| (\tilde{D}_d - Y_{120}'\varepsilon_d), \end{aligned} \quad (8)$$

where $k' \equiv ke^{-i\phi_r}$, $Y_{120}' \equiv V_d^{R\dagger} Y_{120} V_d^{L*}$ and $V_u^{L^T} V_d^{L*} \equiv V_{CKM}^0 = P_u V_{CKM} P_d$. Since the antisymmetric components in eq. (6) are very small, the right-handed quark mixing matrix $W \equiv V_u^{R^T} V_d^{R*}$ can be estimated perturbatively [19]. In the complex case the relevant equation reads ($x = u, d$):

$$W = V_{CKM}^0 + 2(-|\varepsilon_u|Z_u'^* V_{CKM}^0 + |\varepsilon_d|V_{CKM}^0 Z_d'^*)$$

where Z'_x are antihermitean matrices ($M^T = -M^*$) obeying

$$\begin{aligned} Z'_u \tilde{D}_u + \tilde{D}_u Z_u'^* &= e^{i\phi_u} V_{CKM}^{0*} Y_{120}' V_{CKM}^{0\dagger} \equiv A_u \\ Z'_d \tilde{D}_d + \tilde{D}_d Z_d'^* &= e^{i\phi_d} Y_{120}' \equiv A_d \end{aligned} \quad (10)$$

and $\varepsilon_{u,d} = e^{i\phi_{u,d}}|\varepsilon_{u,d}|$. Eqs. (10) are then solved by

$$\begin{aligned} \text{Re}(Z'_x)_{ij} &= \frac{\text{Re}(A_x)_{ij}}{(\tilde{D}_x)_{ii} + (\tilde{D}_x)_{jj}} \\ \text{Im}(Z'_x)_{ij} &= \frac{-\text{Im}(A_x)_{ij}}{(\tilde{D}_x)_{ii} - (\tilde{D}_x)_{jj}} \end{aligned} \quad (11)$$

Having set all relevant notation one can perform a numerical fit for a given set of the additional parameters Y_{120}' , ε_l , $|\varepsilon_{u,d}|$ and $\phi_{u,d}$. We will follow closely the numerical analysis of ref. [19].

A. Electron mass formula and screening of the CKM CP violating phase

Using the hierarchical structure of the RHS of eq. (8) one can again expand the magnitude of the normalized electron mass in powers of λ :

$$|k' \tilde{m}_e| e^{i\phi} = T_{\text{MM}} + \Delta T_{120} \quad (12)$$

Where the symbol T_{MM} stands for the minimal model contribution (the RHS of eq. (4)). The correction coming from the additional terms in eq. (8) reads

$$\Delta T_{120} = -\frac{|r|}{F_s} e^{i\beta_1} F_{\varepsilon_d}^2 [(Y'_{120})_{12}]^2 \lambda^5 + \mathcal{O}(\lambda^6). \quad (13)$$

where $F_s \equiv \frac{m_s}{m_b}/\lambda^3$ and $F_{\varepsilon_d} \equiv \varepsilon_d/\lambda^4$ are $\mathcal{O}(1)$ form factors [19].

Notice that the $\mathcal{O}(\lambda^5)$ term of ΔT_{120} is in general larger than the Λ^2 term on the RHS of eq. (4) and thus the partial cancellation of the leading $\mathcal{O}(\lambda^4)$ in eq. (12) is more easily achieved. As a consequence, one may expect the CKM phase not to be biased towards unphysical values, as it happens in the minimal setup.

By inspection of the $\mathcal{O}(\lambda^5)$ term in ΔT_{120} and the leading $\mathcal{O}(\lambda^4)$ term in (4), a possible way to make these two terms interfere destructively is by taking purely imaginary entries of the Yukawa matrix Y'_{120} , while assuming no spontaneous CP violation. This particular form of the coupling is actually obtained in ref. [24] via an additional parity symmetry that forces all Yukawa interactions to be hermitian. We are now ready to discuss the numerical results.

B. Numerical results

In analogy with the discussion of the CP conserving case we present a sample of the numerical outcomes for a given set of the 120_H parameters. According to the previous discussion we take

$$Y'_{120} = i \begin{pmatrix} 0 & 1 & -1 \\ . & 0 & 1 \\ . & . & 0 \end{pmatrix},$$

$$|\varepsilon_d| = 10^{-3}, \quad |\varepsilon_u| = 10^{-4}, \quad \phi_{u,d} = 0,$$

and $\varepsilon_l = 0$ for simplicity.

In Fig. 5 we display the relative densities of the charged lepton mass solutions as a function of the standard CKM phase. As expected, the tension driving the CKM phase to the second or third quadrant is almost completely screened by the new 120_H induced terms, making the physical δ_{CKM} a natural outcome of the numerical scan. Correspondingly, we find that the presently allowed ranges for the solar and atmospheric mixing can

FIG. 5: The relative densities of the charged lepton mass solutions are shown as a function of the CKM phase δ in the extended 120_H model, for the setup described in the text and 90% C.L. input data from Table I.

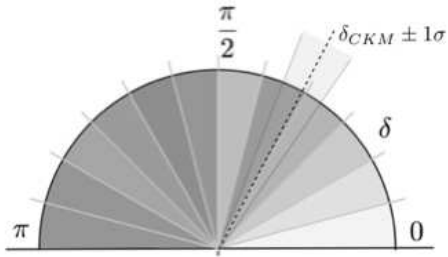
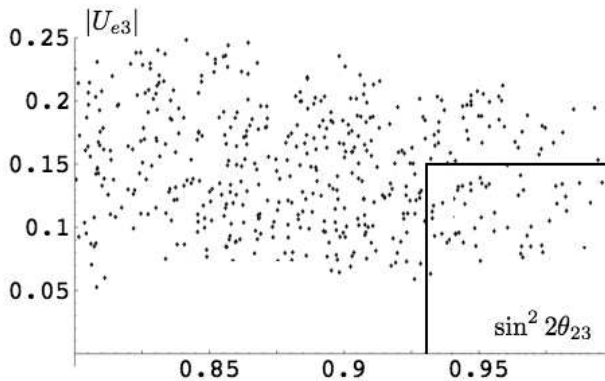


FIG. 6: A typical density plot of $|U_{e3}|$ is shown as a function of $\sin^2 2\theta_{23}$ in the renormalizable SUSY $SO(10)$ model with complex Yukawa couplings and an additional 120_H Yukawa term. The solid contour encloses the experimentally allowed region at the 90% C.L.



be fully covered by tuning the additional contributions. The analogues of Figs. 2 and 3 do not add more information and we omit them here.

It is however worth emphasizing that in spite of the additional freedom in the parameter space a non-vanishing

lower bound remains for the $|U_{e3}|$ mixing angle, as in the CP conserving case [19]. As shown in Fig. 6, obtained for 90% C.L. input data and for the 120_H setup discussed above, we find $|U_{e3}| \gtrsim 0.05$. The bound remains stable for $\tan \beta$ in the 10 to 55 range.

IV. CONCLUSIONS

We have re-examined the role of CP phases within the minimal renormalizable SUSY $SO(10)$ grand unified model, paying particular attention to the fit of the charged lepton masses and the neutrino data. We have shown, against some prejudice present in previous studies, that the observed CP violation in the quark sector can be fully accounted for by the standard CKM phase, without a severe fine tuning on the charged lepton data. While the neutrino Δm^2 ratio covers all of the present 90% C.L. range, both solar and atmospheric angles are obtained in their lower ranges. Finally we emphasize the sharp prediction for $|U_{e3}|$ as a characteristic and robust signature of the model.

As far as the renormalizable extension introduced in ref. [19] is concerned, we have found that the fine tuning in the electron mass is dramatically reduced by the presence of small contributions coming from the additional Yukawa term. The 120_H induced corrections may extend the predictions for the solar and atmospheric neutrino mixings to cover the present experimental ranges. In spite of the additional parameters a robust $|U_{e3}|$ lower bound remains, that characterizes the mass sum rules of the minimal renormalizable $SO(10)$ setup.

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